MATH 17 – QUIZ 4
4 OCTOBER 2010

Name: SOLUTIONS.

NO CALCULATORS

1. Give the general solution to the following differential equations.

(a) $4y'' - 4y' + y = 0$

(b) $y' + \frac{y}{x} = \ln x$

(c) $y'' + 5y' + 6y = 6x^2 - 2x - 2$

a) $4y'' - 4y' + y = 0$

Solve: $4r^2 - 4r + 1 = 0 = (2r-1)(2r-1)$

So the general solution is

$$\boxed{y = c_1 e^{\frac{x}{2}} + c_2 x e^{\frac{x}{2}}}$$

b) $y' + \frac{y}{x} = \ln x$ is a linear first order equation.

Find the integrating factor $e^{\int \frac{1}{x}\,dx} = e^{\ln x} = x$.

So $xy' + y = x\ln x$

$$\int (xy)'\,dx = \int x\ln x\,dx \qquad u = \ln x \quad dv = x\,dx \quad du = \tfrac{1}{x}dx \quad v = \tfrac{x^2}{2}$$

$$xy = \frac{x^2 \ln x}{2} - \int \frac{x}{2}\,dx$$

$$= \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C.$$

So $$\boxed{y = \frac{x\ln x}{2} - \frac{x}{2} + \frac{C}{x}}$$

c) Start by solving the homogeneous equation

$$y'' + 5y' + 6y = 0$$

$$r^2 + 5r + 6 = 0$$

$$(r+2)(r+3) = 0$$

$$y_c = c_1 e^{-2x} + c_2 e^{-3x}$$

Use undetermined coefficients:

$$y_p = Ax^2 + Bx + C$$

$$y_p' = 2Ax + B$$

$$y_p'' = 2A$$

Plug in to get:

$$2A + 5(2Ax + B) + 6(Ax^2 + Bx + C) = 6x^2 - 2x - 2.$$

$$6Ax^2 + (10A + 6B)x + (2A + 5B + 6C) = 6x^2 - 2x - 2.$$

$$6A = 6 \longrightarrow A = 1$$

$$10A + 6B = -2 \longrightarrow 10 + 6B = -2 \rightarrow B = -2.$$

$$2A + 5B + 6C = -2 \rightarrow 2 - 10 + 6C = -2$$

$$\hookrightarrow C = 1.$$

So $\boxed{y = y_p + y_c = x^2 - 2x + 1 + c_1 e^{-2x} + c_2 e^{-3x}}$